Directive radiation from a diffuse Luneburg lens

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Transformation electromagnetics has opened possibilities for designing antenna structures. Using an analytical approach, we demonstrate here how directive antenna radiation can be achieved from an omnidirectional source behind a diffuse surface. This diffuse surface has been obtained by an optical transformation of a Luneburg lens. Two different transformation approaches have been proposed (polynomial and sinusoidal), and for both cases, the resulting material properties have been simplified to ease the fabrication by using all-dielectric media. Therefore, the proposed design has no upper boundary to the operational frequency. Directive radiation has been achieved from thin diffuse structures, which demonstrates promising future possibilities for this technique.

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Recently, the concept of transformation electromagnetic has been demonstrated to be a powerful tool to link the geometry and materials in which the electromagnetic properties remain invariant [1,2]. This technique can be used to reduce the size of conventional devices [3,4], to improve device properties [5,6], or to modify the geometry of antennas which could be originally complex [7,8].

In this Letter, we propose the use of transformation electromagnetics to design new inhomogeneous media which can enable directive radiation from an omnidirectional antenna source behind diffuse surfaces. Here, we propose an example structure whose starting point is a Luneburg lens [9]. This type of lens has the ability to direct the fields in the opposite direction of arrival. Moreover, ideally, Luneburg lenses gradually match any external electromagnetic field arriving from outside of the lens (avoiding any reflection). Despite these impressive properties, however, Luneburg lenses often have not been employed for practical applications because they are restricted by their shape and volumes. Some authors have proposed several modifications to avoid these practical difficulties [7,10–13], but some of the solutions made use of metamaterials resulting in a degradation of the performance, that is, reduction of the bandwidth of operation and an increase in the losses [7,11,12].

In this work, we show the performance of a transformed Luneburg lens which has a size compression in one of its dimensions (being the lens more planar than in the original conception); in addition, its shape has been modified by using a polynomial and sinusoidal pattern that would suit practical conformal surfaces. To make the design feasible, simplifications have been made to achieve all-dielectric diffuse lenses with only materials possessing electrical permittivities that are higher than one and magnetic permeabilities equal to one. Therefore, the proposed implementations do not have limitations in bandwidth.

Luneburg lenses can be implemented as a dielectric sphere in which its permittivity changes with the space position according to the following expression [4]:

$$\epsilon_r = 2 - \left(\frac{r}{R}\right)^2, \tag{1}$$

where r is the position in spherical coordinates and R the radius of the lens.

We next present two examples of diffuse structures (polynomial and sinusoidal), which are designed based on transformation electromagnetics and are capable of producing directional radiation just like those from conventional Luneburg lenses. We first assume a polynomial transformation as follows:

$$x' = \frac{x - R_a + (a \cdot y)^3}{\delta}, \tag{2}$$

$$y' = y, \tag{3}$$

where x' and y' are the transformed space coordinates of the original Cartesian x and y; R_a and a are parameters that define the shape of the new Luneburg lens, and δ is the factor of compression of lens in the x axis. Therefore, according to the theory of transformation electromagnetics [3,14], a distribution of the electric and magnetic material properties that maintain the same electromagnetic performance in the transformed space should be

$$\epsilon' = \frac{J \epsilon J^T}{|J|}, \tag{4}$$

$$\mu' = \frac{J \mu J^T}{|J|}, \tag{5}$$

where J represents the Jacobian transformation tensor. Therefore, after this transformation, the new permittivity distribution that has the same electromagnetic properties follows the expression

$$\epsilon'_r = \epsilon_r \begin{pmatrix} \frac{1}{\delta}(1 + 9a^6 y^4) & 3a^3 y^2 \\ 3a^3 y^2 & \delta \end{pmatrix}, \tag{6}$$

where:

$$\epsilon_r = \left(2 - \frac{y'^2 + (\delta x' + R_a - (a \cdot y)^3)^2}{R^2}\right). \tag{7}$$

Theoretically, the same relation has to be applied to the magnetic permeability (μ_r). The validity of some

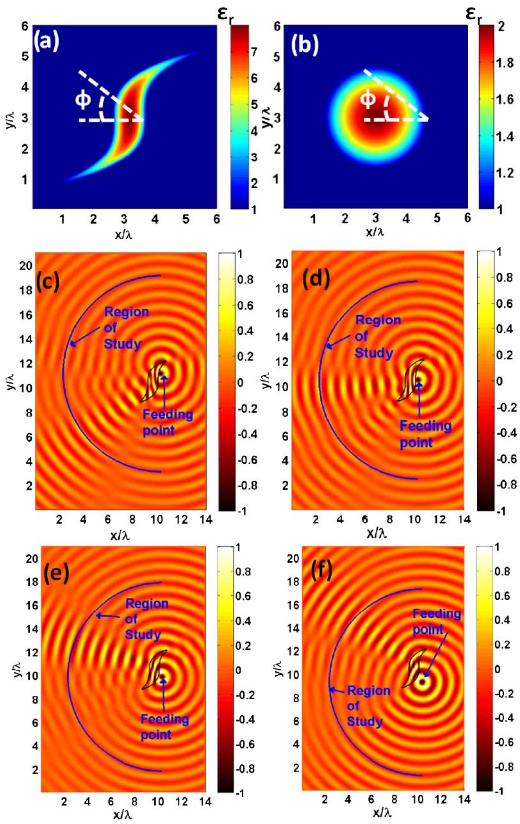

Fig. 1. (Color online) Slim diffuse Luneburg lens (polynomial approach): (a) two-dimensional (2D) permittivity map following Eq. (6) when $R = 1.66\lambda$ with $a = 1/10$, $\delta = 4$, and $R_a = 2R$; (b) 2D permittivity map of Luneburg lens before transformation; and (c)–(f) normalized field distribution in the lens when it is fed at different positions with a normal line source.

simplifications, however, was demonstrated previously [4]. Particularly, it can be assumed that $\epsilon_{xx} = 1$ (to avoid dispersive materials with $\epsilon_r < 1$) and $\mu_r = 1$ (to avoid magnetic materials). A similar simplification, assuming that $\epsilon_{xy} = \epsilon_{yx} = 0$, can be applied to facilitate the practical implementation. For a particular case of a Luneburg lens of size $R = 1.66\lambda$ with $a = 1/10$, $\delta = 4$, and $R_a = 2R$, the obtained dielectric constant that emulates the initial lens is represented in Fig. 1(a). In Fig. 1(b), the original Luneburg lens before transformation is also illustrated for comparison. The field distribution is shown in Figs. 1(c)–1(f) after applying a normal line source for different feeding positions. This calculation was carried out by using an in-house finite-difference time-domain simulator as described in [15,16]. Depending on its position, the electromagnetic fields are directed in different directions of the space, as in a traditional Luneburg lens. To quantify the radiated fields, Fig. 2 shows the normalized field at a distance of 8λ from the source for the positions illustrated in Figs. 1(c)–1(f).

Another feasible transformation to achieve a diffuse surface for directive radiation can be obtained by using a sinusoidal function as follows:

$$x' = \frac{x - R_a + a\,\cos(2\pi \cdot y/R_a)}{\delta}, \tag{8}$$

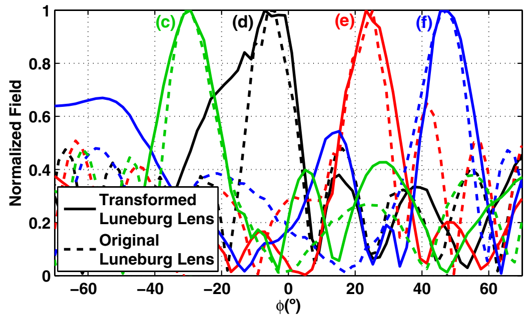

Fig. 2. (Color online) Normalized field distribution for different feeding positions. This field was obtained at a distance of 8λ as it is indicated in Figs. 1(c)–1(f) with a solid blue curve. The results for the original Luneburg lens before transformation are also represented (with dashed curves) for comparison.

$$y' = y. \tag{9}$$

As it was derived before, the dielectric constant for this new space is the following:

$$\epsilon_r' = \epsilon_r \begin{pmatrix} \frac{1}{\delta}\left(1 + \frac{4\pi^2}{R_a^2}\sin^2\left(\frac{2\pi y}{R_a}\right)\right) & -\frac{2\pi}{R_a}\sin\left(\frac{2\pi y}{R_a}\right) \\ -\frac{2\pi}{R_a}\sin\left(\frac{2\pi y}{R_a}\right) & \delta \end{pmatrix}, \tag{10}$$

where:

$$\epsilon_r = \left(2 - \frac{y'^2 + (\delta x' - R_a + a\,\cos(2\pi \cdot y/R_a))^2}{R^2}\right). \tag{11}$$

For the particular case of $R = 1.66\lambda$, and with the parameters of $a = 10$, $\delta = 5$, and $R_a = R$, the obtained transformed dielectric constant is illustrated in Fig. 3(a). Again, we have assumed the same simplifications as in

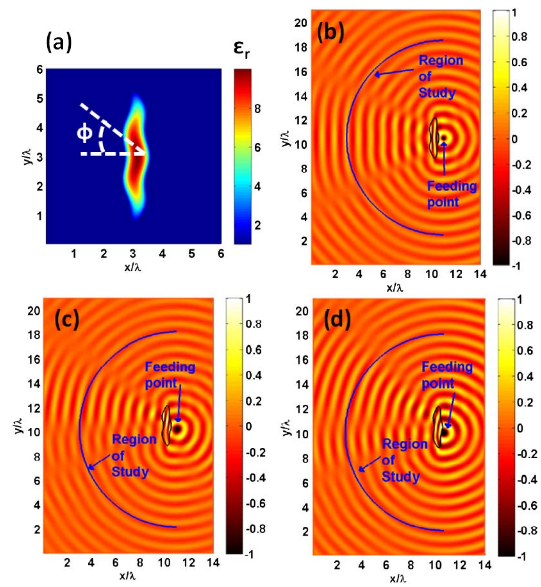

Fig. 3. (Color online) Slim diffuse Luneburg lens (sinusoidal approach): (a) 2D permittivity map following Eq. (10) when $R = 1.66\lambda$ with $a = 10$, $\delta = 5$, and $R_a = R$. (b)–(d) Normalized field distribution in the lens when it is fed at different positions with a normal line source.

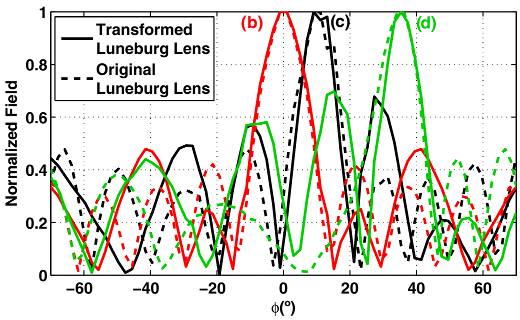

Fig. 4. (Color online) Normalized field distribution for different feeding positions. This field was obtained at a distance of 8λ as it is indicated in Figs. 3(b)–3(d) with a solid blue curve. The original Luneburg lens is plotted with a dashed curve, for comparison.

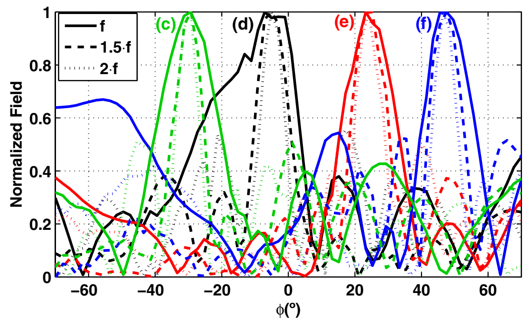

Fig. 5. (Color online) Normalized field distribution for different feeding positions and different operational frequencies for the case of polynomial transformation.

the polynomial transformation to have only conventional materials. The results for this configuration from different positions of the feeding are illustrated in Figs. 3(b)–3(d). As in the previous polynomial transformation, the field (which is excited in the right side of the lens with a normal line source) produces on the left side different directive beams, depending on the location of the feeding: that is, the expected outcome of a traditional Luneburg lens. Figure 4 presents the normalized field distribution in a circumference placed 8λ distance from the source, as indicated in Figs. 3(b)–3(d) for diverse feeding positions; here again, the directive properties of the slim diffuse lens are demonstrated.

The lenses proposed in this Letter have an ultrawide band response, because the dielectric materials required for their fabrication are practically dispersion free and, therefore, do not impose an upper limit to the operational frequency. This is demonstrated in Fig. 5, which illustrates the field distribution for the polynomial lens at the frequency simulated in Figs. 1 and 2 and at 1.5 and 2 times the original frequency. These results illustrate how the increase in frequency produces only an increase of the directivity but not a degradation of the original results. These results are extendable to the sinusoidal approach, although they are not included here for the sake of brevity. As a final note, these lenses will exhibit the conventional limitation at lower frequencies when their size is comparable to the wavelength [13].

In this Letter, we have demonstrated how transformation electromagnetics can contribute to generate directive radiation from diffuse surfaces based on the concept of classic Luneburg lens. To this aim, two different implementations have been proposed: polynomial and sinusoidal space transformations. Both implementations present a considerable size reduction of the original Luneburg lens and a more convenient feeding location. Moreover, they maintain the properties of directive radiation from the original Luneburg lens. Finally, the presented examples are practically feasible with conventional dielectric materials, and they do not have an upper frequency limitation, demonstrating the promising future of transformation electromagnetics in practical applications.

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